

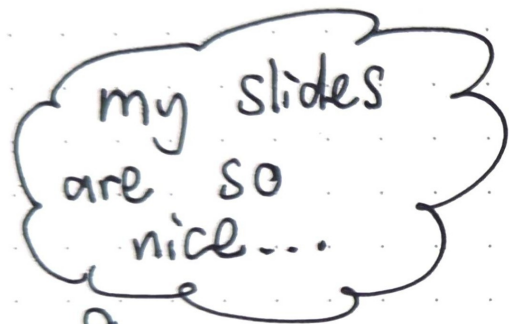
A Lunch Talk About

Spectra

by

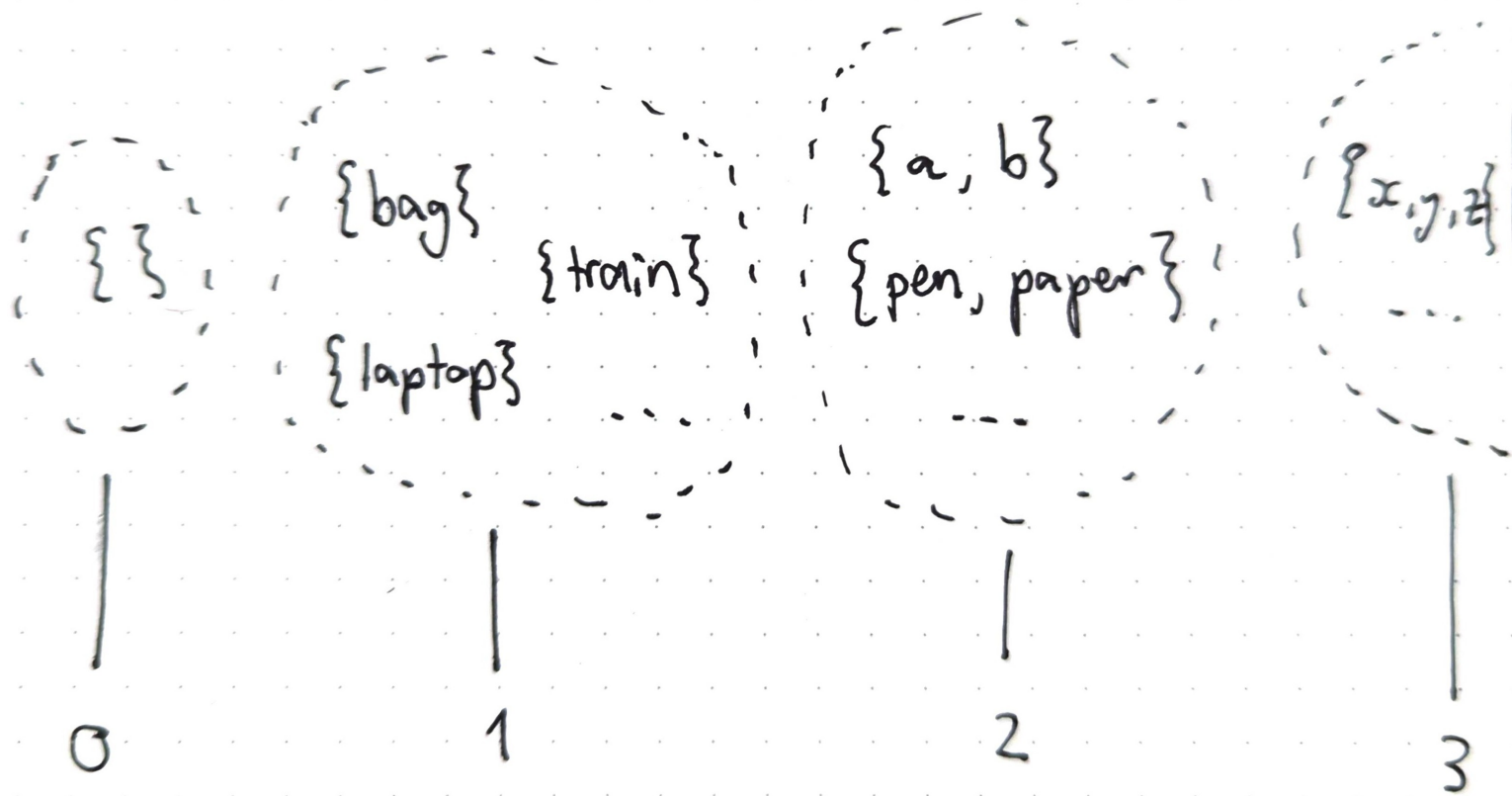
Andres Klone

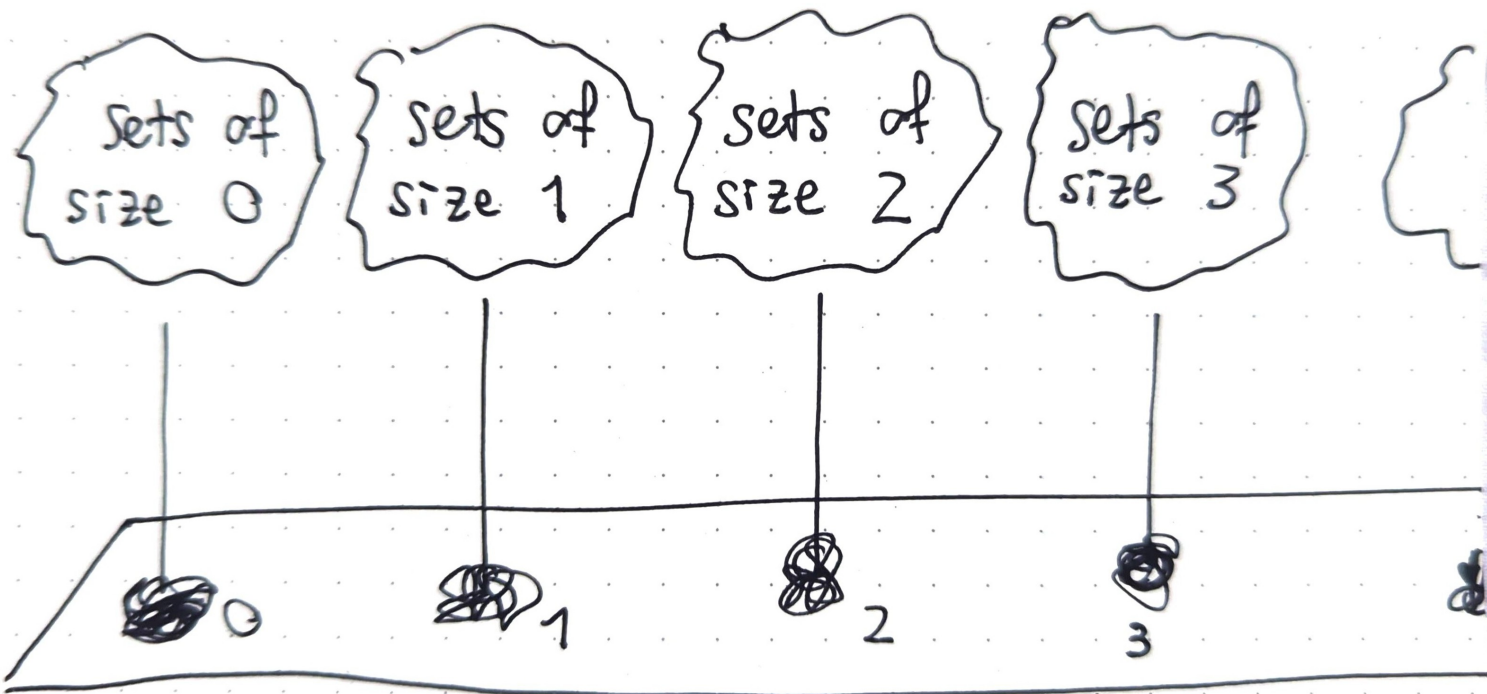
First, a little apology...



idiot

How do numbers arise?





Addition $\longleftrightarrow$ "disjoint union"

$$\{a, b\} \sqcup \{x, y, z\} = \{a, b, x, y, z\}$$

$$\begin{array}{ccccccc} & | & & & | & & | \\ & 2 & + & & 3 & = & 5 \end{array}$$

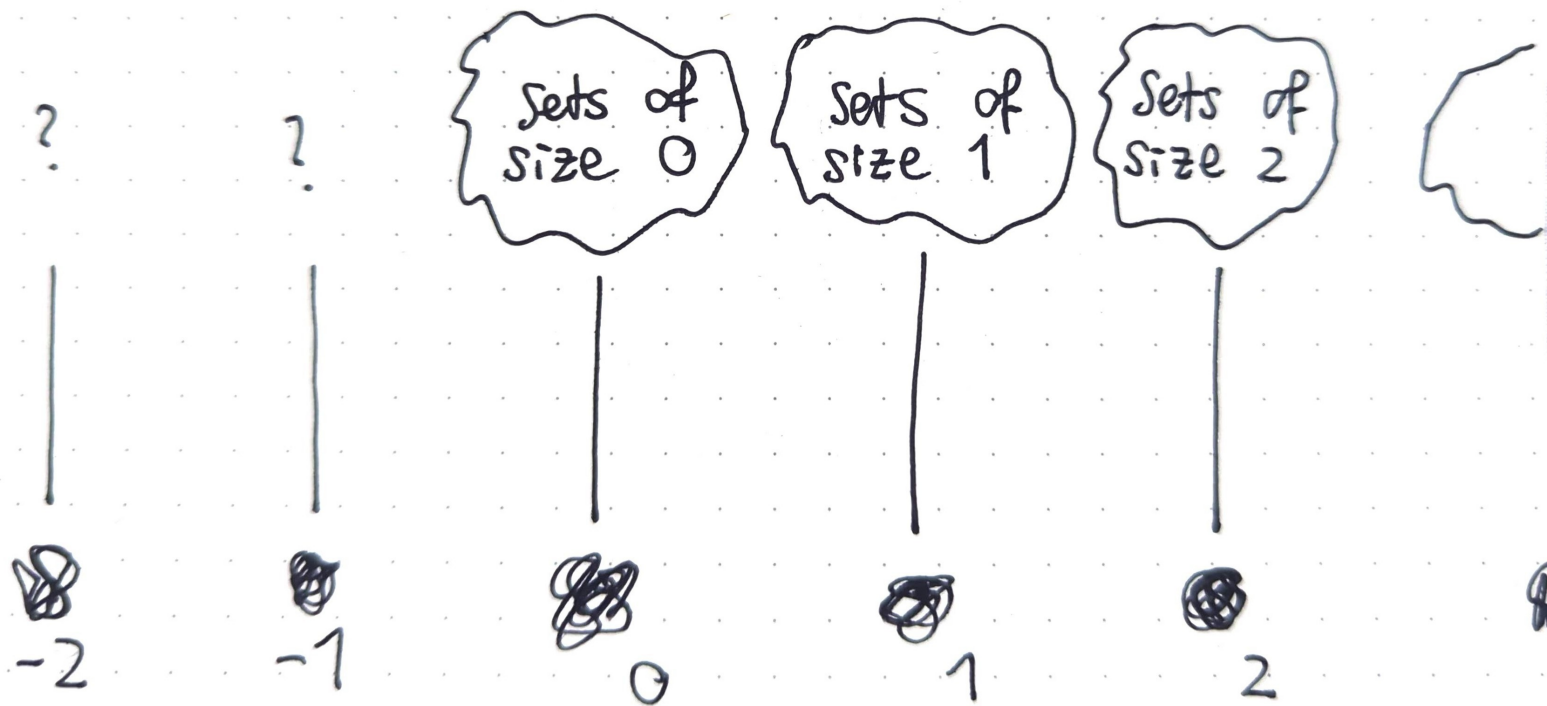
Multiplication $\leftarrow$ "cartesian product"

$$\begin{array}{ccccccc} \{a, b\} & \times & \{x, y, z\} & = & \{(a, x), (a, y), (a, z), \\ & & & & (b, x), (b, y), (b, z)\} \\ \downarrow & & \downarrow & & \downarrow \\ 2 & & 3 & = & 6 \end{array}$$

Slogan

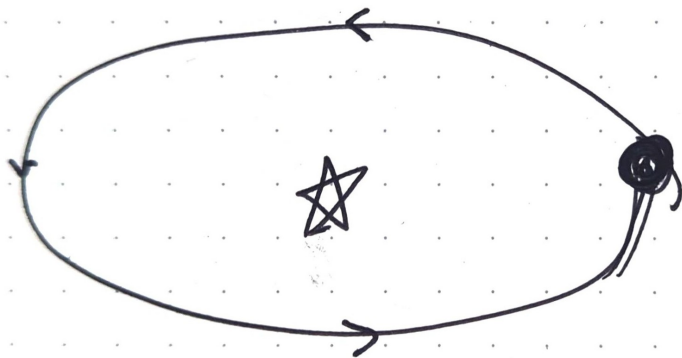
The algebra of numbers arises
from the structure of finite sets.

What about negative numbers?



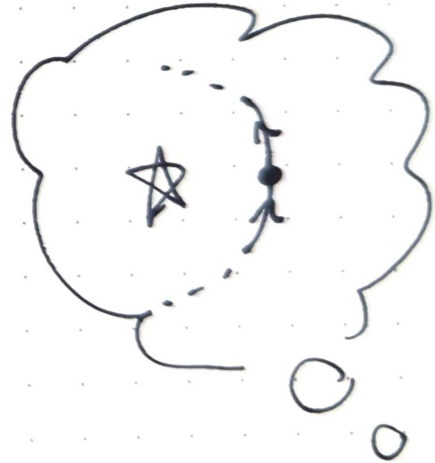
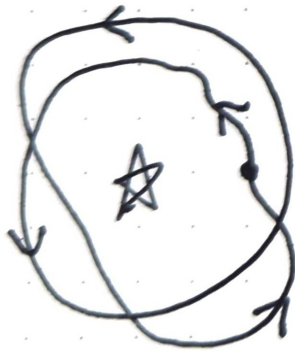
???

Another source of numbers: years

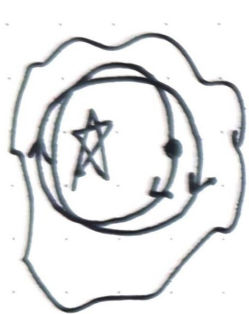


$\dots \rightarrow 2BC \rightarrow 1BC \rightarrow 0AD \rightarrow 1AD \rightarrow 2AD \rightarrow \dots$

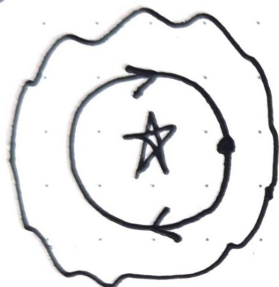
Consider all loops "around" $\star$ (in 2D)



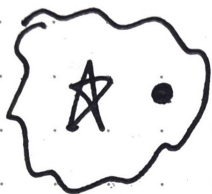
Classify loops by their "winding number"



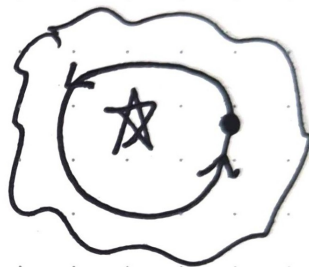
-2



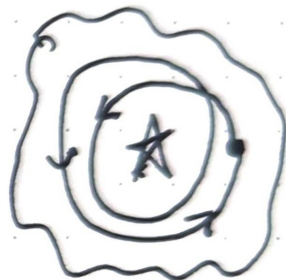
-1



0

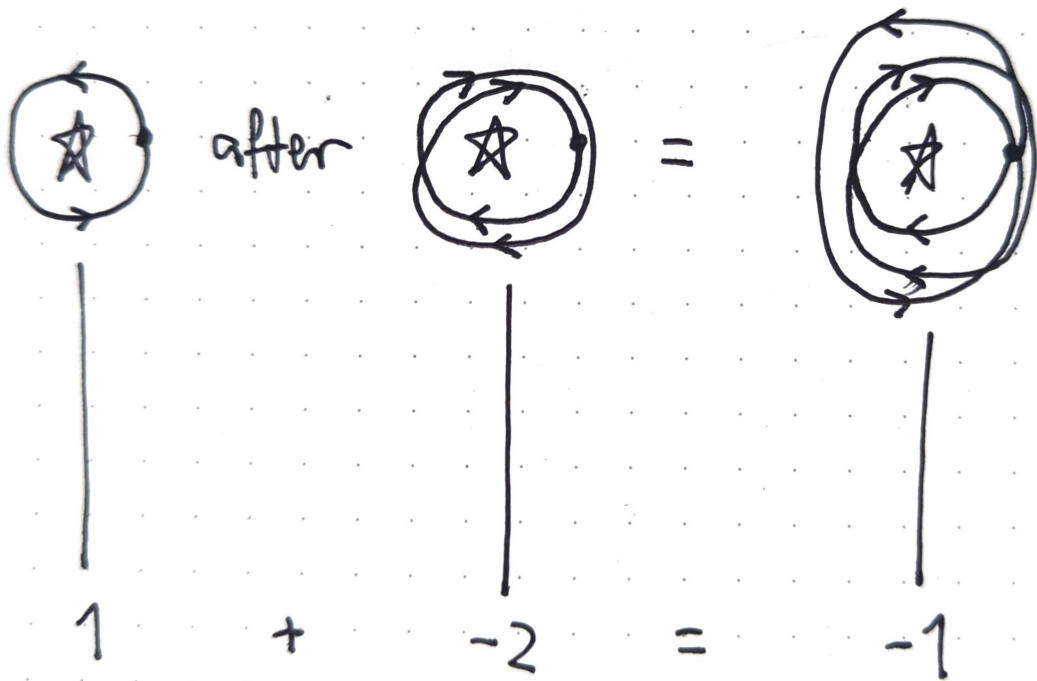


1



2

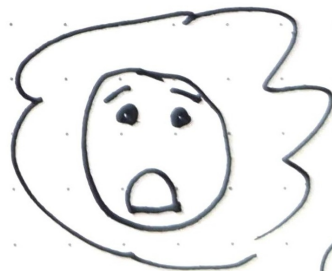
Addition ← "concatenation"



Multiplication $\leftarrow$???

How can I start with loops of "degrees" 2 and 3, and get a loop of degree 6?

You can't.



????

How to make a circle:



Step One:

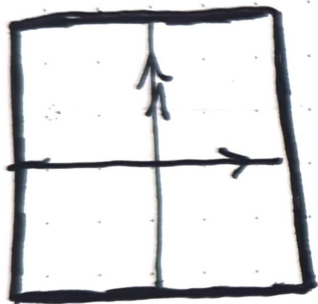


Step Two:

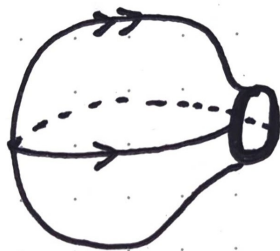


Step Three:

How to make a sphere:



Step One:

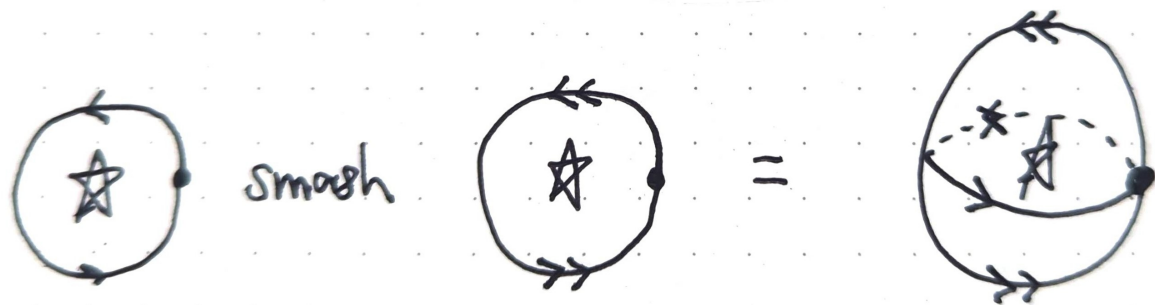


Step Two:

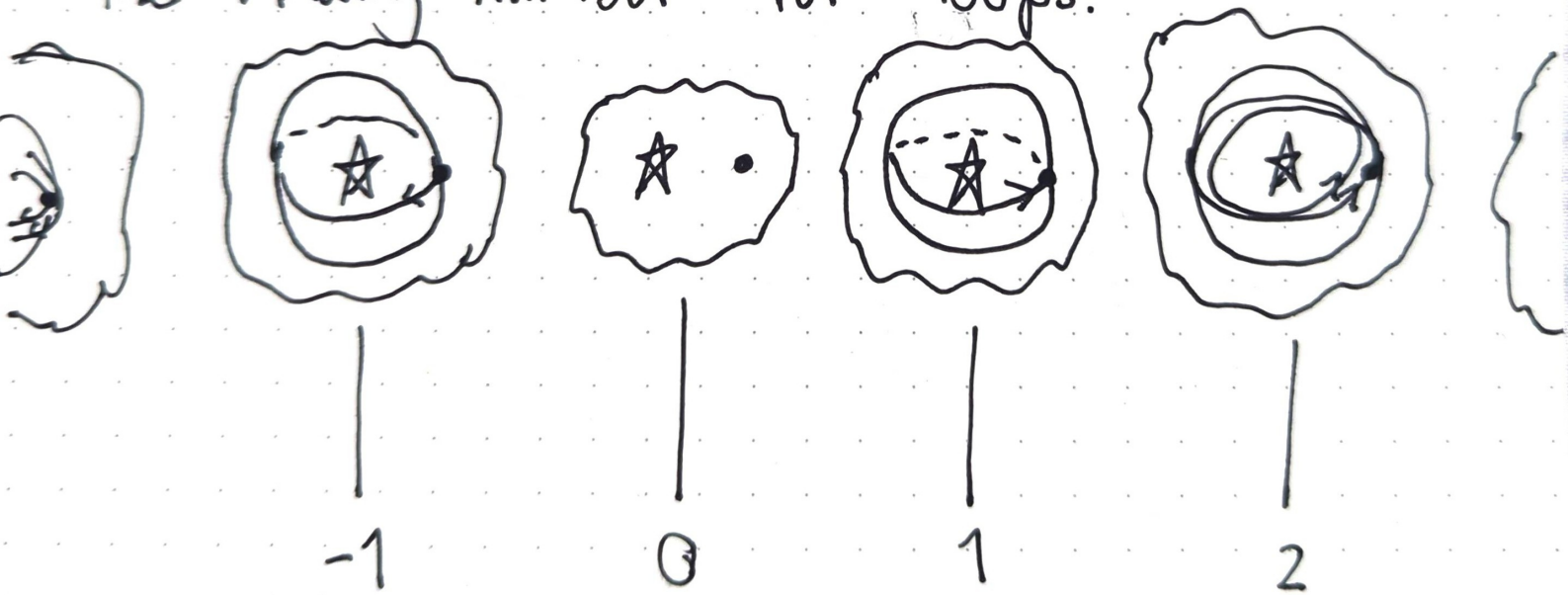


Step Three:

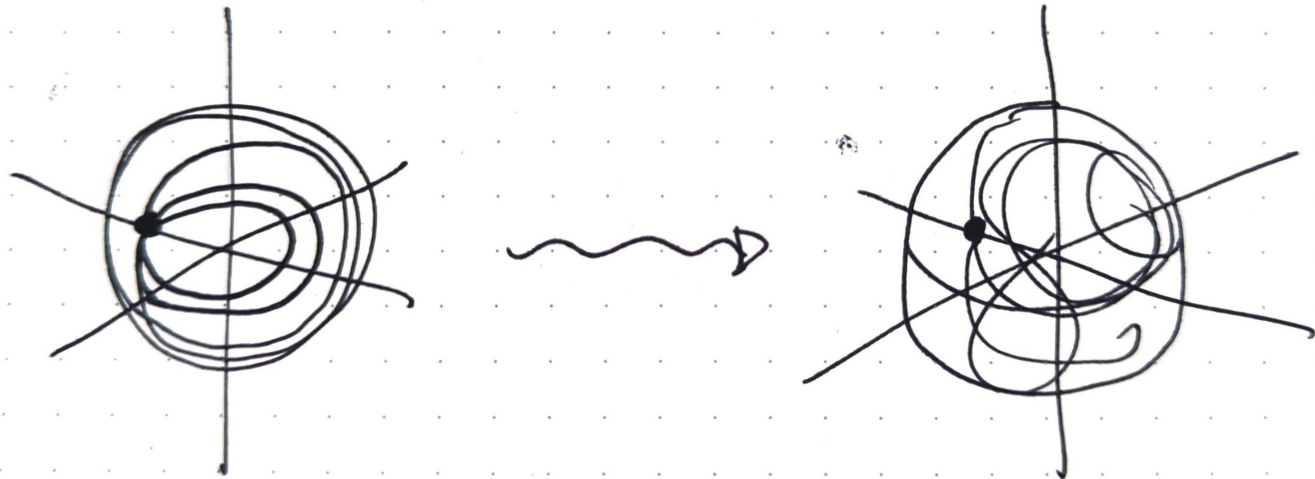
Given two loops around $\star$ in 2D,
we can "smash" them together to get
a sphere around $\star$ in 3D.



All spheres wrapping $\star$ in 3D are classified by an integer, which generalises the "winding number" for loops.

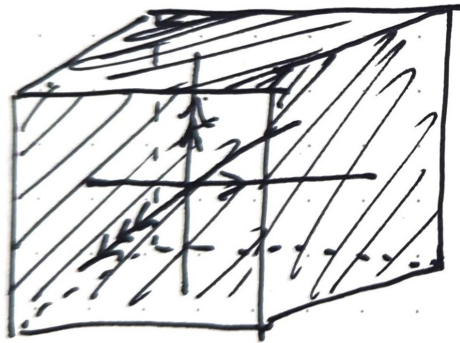


Start with loops of degrees 2 and 3,
and get a sphere of degree 6.



(you will have to believe me)

To multiply three numbers, we need a
"three-dimensional sphere" surrounding
★ in 4D.



Squash
the
boundary
→

To multiply four numbers, we need a four-dimensional sphere surrounding $\star$ in 5D. (... and so on.)

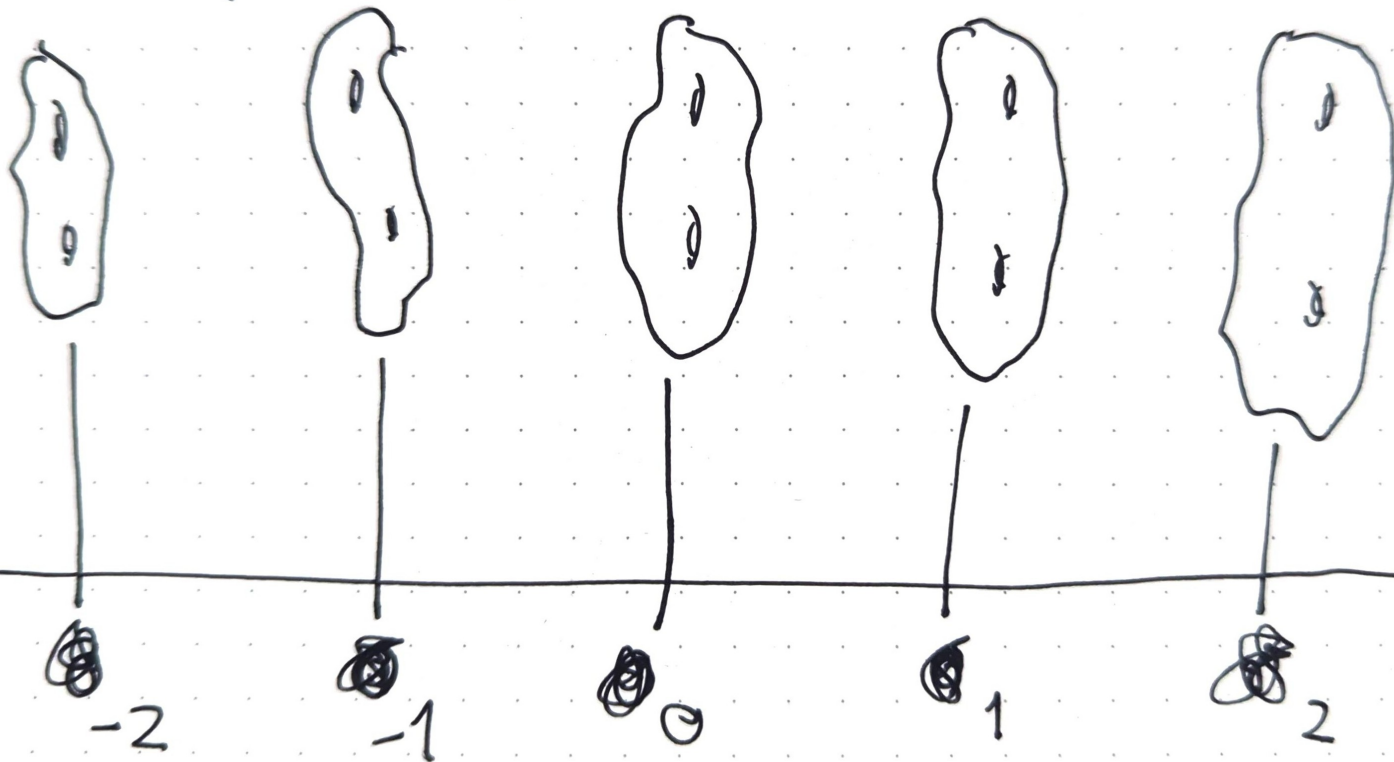
Each of these is classified by its "degree": how many times it wraps around $\star$.

To be able to multiply as many numbers as we want, we need the limit

$$\{\text{loops around } \star\} \leftrightarrow \{\text{spheres around } \star\} \leftrightarrow \{\text{3-spheres around } \star\} \leftrightarrow \dots$$

This limit is the sphere spectrum.

The sphere spectrum:



Slogan

The algebra of integers arises
from the structure of spheres.



Why?



Why?

cf. Ingmar

- 1) For the aesthetic
 - 2) The sphere spectrum is the fundamental example of a derived ring, which are the basic objects in derived geometry
- 

2 continued) The point is that the information we lost in the shadow is genuinely interesting, and important for certain ~~reasons~~ purposes.



3) Many other reasons!

Thank you!